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6. AUTHOR(S)

Charles K. Chui

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Charles K. Chui
Department of Mathematics
Texas A&M University
College Station, TX 77843-3368

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13. ABSTRACT *(Maximum 200 words)*

Abstract. A general framework of scaling functions and wavelets is developed, from which explicit formulas of interpolatory spline-wavelets, compactly supported spline-wavelets, their duals, wavelets on a bounded interval, and trigonometric wavelets are derived. On the other hand, by using the scale of 3 instead of 2, we are able to construct compactly supported orthonormal symmetric scaling functions and their corresponding pair of symmetric and antisymmetric wavelets. To avoid aliasing and other undesirable effects in wavelet decompositions, we introduce a continuous multiresolution analysis that generates the dyadic wavelets of Mallat and Zhong. Another approach is to consider frames. In this regard, we derive Littlewood-Paley inequalities and identities for frames. As an important application, we prove two oversampling theorems: one for generating frames from frames, and the other to insure that tight frames remain tight. For further decomposition of the higher octave bands, wavelet packets are studied and a stability result is obtained. In a different project, we use ridge functions to construct neural networks with one hidden layer, and prove that all such networks only give global approximations. Our study of systems reduction is again based on the AAK approach. We obtained rates of convergence of the rational symbol functions.

14. Cardinal spline-wavelets, spline-wavelets on bounded intervals, trigonometric wavelets, dyadic wavelets, compactly supported symmetric orthogonal wavelets, Bessel families, frames, oversampling, cardinal interpolation, convexity criteria, radial basis functions, ridge functions, neural networks with one hidden layer, Hankel approximation.

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Real-Time Applications

FINAL REPORT

Charles K. Chui
Department of Mathematics
Texas A&M University
College Station, Texas 77843-3368

June 1, 1993

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Table of Contents

I. List of manuscripts submitted or published under ARO sponsorship

Books

1. *Kalman Filtering with Real-Time Applications, Second Edition*, (with G. Chen), Springer Series in Information Sciences #17, Springer-Verlag, 1991.
2. *An Introduction to Wavelets*, Academic Press, Boston, 1992.
3. *Signal Processing and Systems Theory–Selected Topics*, (with G. Chen), Springer Series in Information Sciences #26, Springer-Verlag, 1992.

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II. Participating Scientific Personnel

Principal Investigator
Charles K. Chui

Graduate Students and Research Assistants
Xiaoning Dong
Joerg Hanisch
Dong Hong (Ph. D. August 1993)
Chun Li
Xin Li (Ph. D. May 1991)
J. A. Lian (Ph. D. August 1993)
Ewald Quak
J. Z. Wang

Undergraduate Students
Susan E. Fojtasek
Kathleen Polzer
Sharon K. Zwernemann

III. Brief outline of research findings

The objective of this report is to give a very brief summary of the research findings on the project under Contract Number DAAL03-90-G-0091 sponsored by SDIO/IST and managed by the U. S. Army Research Office. Technical details are not included since most of them have been reported semi-annually. This project convers the period of April 15, 1990-April 14, 1993. The list of publications in Section I, pages 1-3, will be used as references. Other references are listed at the end of this summary.

A general framework of scaling functions and wavelets is developed in [41] from which explicit formulas of interpolatory spline-wavelets [38], compactly supported spline-wavelets, and their duals [39] are derived. By a slight modification, it is also possible to construct other wavelets such as spline-wavelets on a bounded interval [25] and trigonometric wavelets [24]. In addition, a general framework of multivariate wavelets is formulated [15]. Box-spline wavelets are constructed following this framework [35]. On the other hand, by using the scale of 3 instead of 2, we are able to construct compactly supported orthonormal symmetric scaling functions and their corresponding pairs of symmetric and antisymmetric wavelets in [22]. To avoid aliasing and other undesirable effects in wavelet decompositions, we introduce a continuous multiresolution analysis [31] that generates the dyadic wavelets of Mallat and Zhong [S14]. Another approach is to consider frames [S8]. In this regard, we give partial characterizations of frames in terms of Littlewood-Paley inequalities, and dual frames in terms of Littlewood-Paley identities [9, 29]. We also give several equivalent characterizations of frames [29, 30, 34]. As an important application, we prove two theorems on oversampling to generate other frames [33, 29]: one for generating a very large class of frames, and the other to insure that certain frames are tight. For further decomposition of the higher octave bands, wavelet packets are studied and a stability result is obtained [16]. In a different project, we use ridge functions to construct neural networks with one hidden layer [17, 20]. On the other hand, we also prove that all such networks only give global approximations. In other words, when an objective function is locally changed, then all the neurons must be reconstructed in general. For two or more hidden layers, however, we have proved that a local realization scheme can always be constructed. Our study of systems reduction is again based on the AAK approach [S5]. In this direction, we have proved and even obtained the rate of convergence of the rational symbol functions corresponding to the approximations of Hankel operators [18].

In the following, we separate our papers into six categories.

1. **Spline-wavelets**. Our publications in this area are papers [2, 3, 25, 37–41]. The notion of multiresolution analysis was introduced by Mallat and Meyer [S12, S13, and S15]. We gave a general framework along this line of approach [41] and constructed spline-wavelets based on this framework. Among the spline-wavelets we constructed are interpolatory wavelets [38], compactly (and actually minimally) supported spline-wavelets [39], the duals of these wavelets [39], and spline-wavelets on a bounded interval [25]. We also constructed real-time algorithms [2] as well as parallel algorithms [3] for the compactly supported ones and their duals. In addition, we gave a careful analysis [37] of these wavelets, comparing and contrasting with the basic properties of the corresponding cardinal B-splines. For instance, in contrast to total positivity, we introduced the notion of complete

oscillation, which insures a certain amount of zero-crossings. We also constructed certain algorithms and outlined certain computation schemes in [40] for these spline-wavelets.

2. **Other wavelets**. The most well-known wavelets are the compactly supported orthonormal wavelets of Daubechies [S6, S7]. However, as already observed in [S6], these wavelets cannot be made symmetric or antisymmetric unless we settle for the Haar function [S10], which of course, gives very poor time-localization. In signal analysis, however, symmetry or antisymmetry are among the most important properties, since they are essential to linear-phase filtering. To achieve symmetry and antisymmetry, there are several ways to go. The easiest way is to give up compactly supportedness, such as the Battle-Lemarié wavelets [S1, S11] and the Meyer wavelets [S15]. We are not willing to do this since FIR filters are important in real-time applications. The compactly supported spline-wavelets discussed above motivate one approach we took. One of the attractions of this approach is that the decomposed wavelet components remain orthogonal. We call such wavelets semi-orthogonal wavelets in [S3]. Another attraction is that the compactly supported spline-wavelets are very similar to the original wavelets of Morlet [S9], with the advantage that they are much more efficient in implementation. A unified formulation of orthonormal wavelets, semi-orthogonal wavelets, and non-orthogonal (sometimes called bi-orthogonal) wavelets is given in [S3]. Very recently, we found another way to achieve symmetry without giving up compactly supportedness and orthonormality in [22]. The trick in [22] is to change the scale from 2 to 3. In doing so, we are able to construct symmetric scaling functions along with their corresponding pairs of orthonormal compactly supported symmetric-antisymmetric wavelets. This is done in [22] for each preassigned regularity exponent. Another type of wavelets we constructed is a class of trigonometric wavelets [24]. Our motivation was to keep the sinusoidal waveform in wavelet decomposition. In all of the above discussions, we were only concerned with wavelet decompositions using a digital format. In some applications, however, as pointed out by Mallat and Zhong [S14], since the pyramid algorithm requires downsampling (which has the effect of undersampling), this type of algorithm causes undesirable effects, such as aliasing. In [S14], wavelet decompositions which are continuous-time along each scale and discrete-time between scales were recommended. The analyzing wavelets for such decompositions are called dyadic wavelets. In [31], we developed a multiresolution analysis for generating dyadic wavelets, and relate this study to that of the so-called up-functions. On the other hand, in our study of systems theory, we saw a need of certain system decomposition governed by convolution with various scales, and hence, introduced a fairly general bivariate decomposition scheme in [19].

3. **Bessel families and frames**. Dyadic wavelet decompositions are both aliasing-free and translation-invariant. However, the decomposition scheme is very expensive, and definitely cannot be implemented in real-time. In any case, since integration can only be performed numerically by taking summations with very dense discrete samples anyway; it is really equivalent to the consideration of digital algorithms. This motivated our study of frames. The notion of frames was introduced by Duffin and Schaeffer [S8] in the study of nonharmonic analysis. Since we are mainly interested in frames which are generated by dilations and translations, there is much more to gain than the original study in [S8].

In particular, in oversampling, it is sufficient to worry about the upper stable bounds; and this is equivalent to considering Bessel families. In [29], we gave several complete characterizations of Bessel families, and relate this study to that of affine-frame operators. In [32], we showed that such operators are, in fact, of Calderón-Zygmund type [S15]; and hence, we have returned to the mathematical origin of wavelets, namely: the Calderón theory [S2]. One of the most important applications of frames in our approach is the development of two fundamental theorems on oversampling: the first one is for creating frames by oversampling frames (such as Riesz basis), and the second one is to insure tight frames (such as orthonormal wavelets) remain tight after oversampling with frequency $n > 1$. Our result is that tightness is guaranteed if n is relatively prime to the scale a. For example, if $a = 2$, as usually used, then n must be an odd integer; and in fact, examples are given in our paper [33] that even $n = 2$ destroys the tightness of the orthonormal Haar basis.

4. **Tutorial and survey papers**. There has been very high demand from various international conferences and workshops, both in the U. S. and abroad, to lecture on our wavelet results during the past three years. Some of these invitations requested publications of tutorial or survey papers related to the lectures. Papers [6–13, 25, 34] were written and published to meet these requests.

5. **Splines**. Our original motivation to the study of wavelets is their most natural relationship with splines. In fact, the P. I. learned about wavelets by attending an hour-address given by A. Cohen (substituting for Y. Meyer) in the 1989 Army Conference on Applied Mathematics and Computing held in June at West Point. In his lecture, Cohen used a B-spline to demonstrate how an "edge" is localized and detected. As mentioned above, we have built wavelets by using cardinal B-spline functions. In fact, we have even built compactly supported wavelets by using B-splines on a three-directional mesh in [35]. Hence, it is very important for our research program not to leave the investigation on splines. Our spline papers during this funding period are [4, 5, 14, 36]. Because of their intrinsic properties, we were only concerned with the study of shapes (particularly convexity) [4, 5] and computational schemes [14, 36].

6. **Neural networks and related problems**. Recently, Approximation Theory has played on active role in the theoretical development of Systems Theory in engineering [S4, S5]. Our contributions in this area can be classified into three categories: neural networks, AAK theory in H^∞ control, and polynomial/rational approximation. Since this is really not the main theme in our research program, we were only interested in very basic research and did not expect any immediate payoff in terms of applications. This attitude actually helped us greatly in proving beautiful mathematical theorems [26–28] on polynomial interpolation and approximation, and developing new techniques for solving engineering problems, such as [17, 20, 1] for constructing and studying neural networks, and [18, 21, 23] for systems/control theory. In particular, by introducing ridge functions to neural networks, we proved in [17] that one hidden layer is sufficient, even by using only integer shifts, to approximate any function in any dimensions; and in [20], we even have a scheme for the realization of such neural networks. In H^∞-control, our paper [18]

is probably most fundamental, in that it gives an elegant formulation of an outstanding problem in the AAK theory and solves this problem under this formulation. As a result, stable finite-rank systems can be realized to replace very complicated (even finite-rank) systems with a known error bound, given by the singular values (or s-numbers) of the corresponding Hankel operators.

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The list of publications by the P. I. listed in Section I is used as references in the description of research findings. The following supplementary list is also relevant to the final report.

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